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ABSTRACT

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Factorial ATI Analysis

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Abstract

An approach to the analysis of an aptitude-treatment-interaction (ATI) design in which the treatment groups are based on an underlying factorial structure was described and illustrated. The approach emphasized description with point and interval estimation. The example design considered consisted of two nominal treatment variables and one interval aptitude variable. Contrasts analogous to various effects in three-way ANOVA were identified for the description of the ATI.

Factorial ATI Analysis

Cronbach (1975) has argued that the currently prevalent research strategy based on hypothesis testing is wasteful for the study of interactions among instructional treatments and student aptitudes. This strategy dictates that only those effects which are statistically significant should be included in final models and descriptions. Power calculations in Cronbach and Snow (1976, pp. 55-57), however, indicate that typical instructional research studies of moderate size (with, say, 40 or 50 observations per cell) clearly have inadequate power for interaction hypotheses; thus, evidence for interactions of practical significance may be overlooked, resulting in oversimplified descriptions. This problem is aggravated in studies with limited resources where the class or some larger aggregate is the appropriate unit of analysis.

An alternative strategy suggested by Cronbach is to "exorcise the null hypothesis" and concentrate on the description of interaction effects with point and interval estimation. The point estimate of a contrast of interest represents a single best estimate of the true value and the width of the confidence interval about the point estimate makes the precision of the estimate explicit. Of course, point and interval estimation of contrasts is recommended in standard texts (e.g., Kirk, 1968) as part of the follow-up for hypothesis testing. The point is that study results should be described as directly and completely as possible without interposing a hypothesis testing screen which may result in an oversimplified model. The resulting relatively complex

descriptions will probably contain features due only to sampling error, but consistencies which emerge over a series of quasi-replications will hopefully form the basis of reliable interactive models.

This paper demonstrates such a descriptive approach for a relatively complex aptitude-treatment-interaction (ATI) design. A recent elaboration in ATI studies which consider one or more aptitudes represented by interval scales (i.e., continuous aptitudes which have not been categorized to form nominal factors) is the use of treatment groups based on an underlying factorial structure (e.g., Mayer, 1975). Suppose, for example, a researcher wishes to study the interactions between the instructional dimensions of "extent of structure" and "type of testing" and the student aptitude of general ability. A possible ATI study might consider six treatment groups formed by crossing the structure factor with two levels and the testing factor with three levels. The description of results associated with such a design which is illustrated in this paper includes features analogous to the usual description in a three-way ANOVA with all nominal factors; the required analysis and presentation formats, however, are different from those for three-way ANOVA. The analysis used the REGRESSION program from the SPSS computer package (Nie, Hull, Jenkins, Steinbrenner, & Bent, 1975) and a relatively simple program created to compute interval estimates.

Example Analysis

Assume that a factorial ATI design has two nominal treatment factors, factor B with two levels B1 and B2 and factor C with three levels C1, C2, and C3. The continuous student aptitude will be called A. A researcher

might randomly assign classrooms from a population of interest to each of the six treatments resulting from the crossing of factors B and C. Thus, treatment or cell jk is based on level j of factor B and level k of factor C. The aptitude of each student would be measured, the treatments would be administered to classes, and an outcome such as student performance, attitude, etc., measured. Since classes are the sampling units, class averages of aptitude and outcome would be used for analysis.

The analysis is based on the fixed-effects linear model (e.g., Graybill, 1976), i.e., $Y_s = \mu_s + \varepsilon_s$, where Y_s is the observed dependent variable for the s^{th} analysis unit ($s = 1, \ldots, n$), μ_s is the expected value of Y for the set of v independent variables X_{st}, $t = 1, \ldots, v$, given by $\mu_s = \beta_0 + \beta_1 X_{s1} + \beta_2 X_{s2} + \ldots + \beta_v X_{sv}$, and ε_s is the residual assumed to be normally and independently distributed with mean zero and variance σ^2. Any nominal independent variable with r levels can be represented in the model by a set of r-1 dummy variables. For example, using 1,0 coding, a unit in any level $u \leq r-1$ receives a 1 for the u^{th} variable in the set of dummy variables and 0's for the other variables. Units in level $u = r$ receive 0's for all variables. Interactions between interval and/or nominal independent variables are represented in the model by products of the main effect terms. Thus, in the current example with one aptitude (A) and the two factors B and C, the three two-way interactions (BC, AB, and AC) are represented by the sets of possible pairwise products across two variables and the ABC three-way interaction by the set of possible triple products across the three variables. The specification of such models for mixed nominal and interval independent

variables has been discussed, for example, by Cronbach and Snow (1976, pp. 71-73) and Kleinbaum and Kupper (1978, pp. 197-199). For the current example, the resulting coding for each unit in, say, cell 12 of the 2x3 layout would be the following: X_1 = unit score on A; X_2 = 1 for B main effect; X_3 = 0 and X_4 = 1 for the C main effect; X_5 = 0 and X_6 = 1 for the BC interaction; X_7 = A for the AB interaction; X_8 = 0 and X_9 = A for the AC interaction; and X_{10} = 0 and X_{11} = A for the ABC interaction.

Study results can be described with various linear combinations of the model parameters (β's). Assume a specific combination is defined as $\gamma = \underset{\sim}{g}'\underset{\sim}{\beta}$ where $\underset{\sim}{\beta}$ is the column vector of parameters $\beta_1, \ldots, \beta_v$ and $\underset{\sim}{g}'$ is the row vector of defining coefficients. For example, a researcher may wish to consider the difference in the expected value μ at levels C_2 and C_3 of factor C for level B1 of factor B and an aptitude equal to A_i. If μ_{ijk} is defined as the population value of μ in cell jk for A_i, then the contrast of interest is $\mu_{112} - \mu_{113}$. Regression equations of μ on A for each cell, shown in Table 1, can be found by substituting the appropriate coding and rearranging terms; the resulting expressions can then be substituted into the contrast of interest. The contrast $\mu_{112} - \mu_{113}$ can be shown with this approach to be equal to $(\beta_4 + \beta_6) + (\beta_9 + \beta_{11})A_i$ in the population; that is, the coefficient vector $\underset{\sim}{g}'$ is equal to $(00010100A_i0A_i)$. The sample point estimate of this contrast is obtained with the least squares estimates of the β's.

Insert Table 1 about here

The 1-α confidence interval on γ has 1-α probability of capturing the true value of γ and is computed with $CI(\gamma) = \hat{\gamma} \pm CS_{\hat{\gamma}}$, where the estimated standard error of the estimator $\hat{\gamma}$, $S_{\hat{\gamma}}$, is equal to $[\underset{\sim}{g}' \, V(\hat{\beta}) \, \underset{\sim}{g}]^{\frac{1}{2}}$. $V(\hat{\beta})$ is the estimated variance-covariance matrix of the estimators $\hat{\beta}_1, \ldots, \hat{\beta}_v$ given in the output of the SPSS package. A simple 1-α confidence interval for a specific γ is computed with $C = t(1-\frac{\alpha}{2}; n-v-1)$. Potthoff (1964), however, has argued that it is often more appropriate to consider simultaneous confidence intervals, i.e., a family of intervals computed in such a way that the probability is 1-α that all of the intervals will capture their respective parameters. Simultaneous intervals would appear to be particularly appropriate in the study of interactions where complex patterns will be analyzed into many contrasts to be considered simultaneously. The Scheffé method (Graybill, 1976, pp. 195-200) is one approach appropriate for large families of complex contrasts. If a family is defined to be all of the infinite number of linear combinations of a subset of, say, h β's from the entire model, the appropriate critical value is $C = [h \, F(1-\alpha; h, n-v-1)]^{\frac{1}{2}}$. Example calculations below include both simple and simultaneous intervals to illustrate the cost in interval width associated with the simultaneous protection.

Hypothetical data (n = 48) for the analysis were generated by assuming $\beta_0 = 2$, $\beta_1 = .8$, $\beta_2 = 0$, $\beta_3 = 4$, $\beta_4 = 6$, $\beta_5 = 2$, $\beta_6 = 3$, $\beta_7 = -.8$, $\beta_8 = .4$, $\beta_9 = -.6$, $\beta_{10} = -.6$, and $\beta_{11} = .8$. Eight values of Y were obtained for each of the cells in the 2x3 design by adding and subtracting one from the computed value based on the coefficients above with the

aptitude equal to 2, 4, 6, and 8 in turn. For example, the computed value of μ for cell 11 with aptitude equal to 2 is 7.6, resulting in two Y "observations" of 8.6 and 6.6.

Analysis of the data using the SPSS Regression program resulted in β estimates which, due to the symmetry of the artificial data, are identical to those defined above. The resulting six regression equations shown in Figure 1 represent the overall effect of the three independent variables; the vertical distances between the lines represent the effects of the nominal factors B and C, the slopes of the lines represent the effect of the aptitude A, and the slope differences reflect the aptitude-treatment interaction. The overall R^2 of .934, $F(11, 36) = 46.4$, $p < .001$, and the R^2 increment of .076 due to the ATI terms, $F(5, 36) = 8.29$, $p < .001$ are both of practical significance.

Insert Figure 1 about here

The goal is the description of effects and interactions associated with the three independent variables. Such description would involve, at the most detailed level, the effect of a variable at specific levels of the other two variables and the interaction between two variables at a specific level of the third variable; these effects are usually called a simple simple main effect and a simple interaction, respectively, in treatments of three-way factorial ANOVA. This detailed description may sometimes be simplified when one or more interactions are not of practical importance. It may then be appropriate to consider the effect of one

variable at a specific level of a second variable averaged over all levels of the third variable (simple main effect), the effect of a variable averaged over all levels of the other two variables (main effect), and the interaction of two variables averaged over levels of the third variable.

All information necessary for the point estimation of any effect of interest is contained in Figure 1, and some effects can be easily identified. For example, the simple simple main effect of A at B_jC_k is just the slope of the regression line for cell B_jC_k. Most other effects, however, are not as easy to identify with the format of Figure 1. Two more descriptive formats are described below; the first, called ATI format because it is based on various groupings of the Y on A regression lines, effectively describes the B and C simple simple main effects and the AB and AC simple interactions. The possible simplification of the description is also considered. The second format considered below clarifies the nature of the BC interaction.

ATI Format. Different groupings of the regression lines in Figure 1 are formed, one grouping for each row and column in the 2x3 layout for factors B and C. For example, the regression lines for treatment cells 21, 22, and 23 are shown in Figure 2. The vertical distances between the lines for a specific A_1 represent the simple simple main effect of C at A_1 and B2, and the different slopes in the pattern reflect the simple interaction of AC at B2.

Insert Figure 2 about here

The degree of confidence which can be placed in any of the features of the pattern in Figure 2 can be established with confidence intervals on appropriate contrasts. Note, for example, that a prominent feature is the disordinal interaction associated with treatments 21 and 22. Treatment 22 is estimated to be superior up to an aptitude of approximately two, while treatment 21 is superior for higher aptitudes. The confidence that this feature in the sample reflects a similar feature in the population can be determined with the consideration of one or both of two contrasts. One represents a component of the simple interaction of AC at B2, i.e., the difference in slopes of the lines for cells 21 and 22. Substitution from Table 1 results in $S_{21} - S_{22} = \beta_8 - \beta_9$. The 90% simple confidence interval for this contrast is equal to $1.0 \pm .436$ based on a standard error of .258 and $C = 1.69$. The computation of a simultaneous confidence interval on the same contrast requires that an associated family of contrasts be defined. It can be seen from Table 1 that all possible pairwise slope comparisons for the cell equations would involve only the coefficients β_7 through β_{11}, i.e., those coefficients for the independent variables representing the two-way and three-way interaction terms involving A. If the family is defined as all linear combinations of β_7 through β_{11} (i.e., if h is 5), the resulting value of C is 3.17, and the interval half-width is .818, nearly twice as large as the simple interval half-width. Thus, there is 90% confidence that all intervals in the defined family (including 90% CI($S_{21} - S_{22}$) = $1.0 \pm .818 = .082$, 1.818), capture their respective parameters. In this case, both the simple and simultaneous intervals for $S_{21} - S_{22}$ are relatively wide,

indicating the point estimate is not precise; the intervals are narrow enough, however, to allow confidence in a conclusion about the direction of the slope difference.

A second contrast, reflecting a component of the simple simple main effect of C at A_1 and B2, is also useful in establishing the degree of confidence in the same feature. The vertical difference between the lines for cells 21 and 22 at a specific aptitude is determined from Table 1 to be $\mu_{121} - \mu_{122} = (\beta_3 - \beta_4) + (\beta_8 - \beta_9)A_1$. This contrast answers common ATI questions: Which treatment is superior for a specific aptitude, how great is the superiority, and how does this picture change with varying aptitude? The point and interval estimates for this contrast are shown in Figure 3 as a function of aptitude. The "family" associated with the simultaneous confidence band might be defined by noting that all such mean contrasts for any value of A can be expressed in terms of the coefficients β_2 through β_{11}. The value of h, then, required to compute the critical value C is 10. The simultaneous confidence band suggests confidence in the superiority of level C1 over C2 (for level B2) for relatively large values of A but also indicates strong uncertainty as to where or even if the superiority reverses for small A values. If desired, these confidence bands can be used to establish "regions of significance" (Johnson and Neyman, 1936) by noting the values of A at which the intervals no longer capture zero. In Figure 2, the simultaneous region of significance ranges from approximately 4.5 to the upper limit of the available data.

Insert Figure 3 about here

The two contrasts considered thus far represent two ways of viewing a feature of interest in Figure 2; the simple interaction contrast represents the feature as a slope difference while the simple simple main effect contrast represents it as a variation of a mean difference with aptitude. Both views provide some insight and would be used to consider other features in the patterns resulting from the five possible groups of regression lines, two for factor B and three for factor C. The ultimate goal of such analysis is the identification of reliable aspects of the sample effects and interactions.

The ATI description discussed to this point will present a detailed and relatively cumbersome picture. Such detail may be necessary if all of the various interactions are of practical importance, but a researcher would want to simplify the description if possible. Consider, for this purpose, "higher order" interaction patterns constructed in the following way. The contrasts of ultimate interest for each of the nominal factors are identified. Assume here that pairwise comparisons are important; thus, there is one comparison comprising the factor B effect and three for the factor C effect. Equations representing a contrast for one factor at each of the levels of the other factor are collected into a single pattern. In this example, there would be four of these contrast patterns, one pattern with three lines for the B1-B2 comparison and three patterns with two lines each for the three pairwise comparisons of the levels of C.

For example, the C1-C2 comparison for the two levels of B is shown in Figure 4.

Insert Figure 4 about here

The pattern in Figure 4 can be analyzed with the same approaches used above for the cell equation patterns, i.e., using slope differences and vertical differences between the lines. Consider first how the possible contrasts may be of interest for their own value before using them to attempt a more parsimonious description. One question, for example, is whether the C1-C2 difference at B1 is more sensitive to A variation than the same difference at B2. The difference in the slopes for the two lines in Figure 4 is given by $S(\mu_{111} - \mu_{112}) - S(\mu_{112} - \mu_{122}) = \beta_{10} - \beta_{11}$. As suggested by the coefficients involved, the question is concerned with the three-way interaction among A, B, and C. If the family for this contrast is considered to be all combinations of β_{10} and β_{11}, a family which would contain all slope differences for the contrast patterns, the resulting simple and simultaneous intervals are $-1.4 \pm .619(.810)$. Thus, the estimated direction of the difference in A sensitivity is reliable.

A second question might be whether the C1-C2 difference at B1 is different from that at B2 at various values of A. The pertinent contrast, which reflects part of the simple interaction of BC at A, is given by $(\mu_{111} - \mu_{112}) - (\mu_{121} - \mu_{122}) = (\beta_5 - \beta_6) + (\beta_{10} - \beta_{11})A_i$. All contrasts of this type for the possible patterns can be expressed in terms of

coefficients associated with the BC and ABC terms, i.e., β_5, β_6, β_{10}, and β_{11}. The resulting 90% simple and simultaneous confidence bands are shown in Figure 5. It is seen that the C1-C2 difference for B2 is reliably greater than that for B1 for most of the A range with the difference becoming larger with increasing A.

Insert Figure 5 about here

Simplification of the ATI description using contrast patterns like that in Figure 4 can be illustrated with some possible configurations for the C1-C2 pattern shown in Table 2. The two lines in the first pattern are nonparallel; in this case, the three-way interaction component associated with this contrast (i.e., $S(\mu_{111} - \mu_{112}) - S(\mu_{121} - \mu_{122})$) is estimated to be of practical importance and the contrast must be described at the simple simple main effect level with results like those in Figure 3. In contrast, the other four patterns all show estimated parallel lines, i.e., they reflect situations in which the three-way interaction component above is estimated to be approximately zero. The two-way interactions associated with the contrast of interest (i.e., AC and BC) should then be considered to decide if simplification is possible. The BC component associated with the C1-C2 contrast is $[(\mu_{.11} - \mu_{.21}) - (\mu_{.12} - \mu_{.22})]$ and the AC component is $S(\mu_{i.1} - \mu_{i.2})$; thus, the BC component indicates the difference in levels of the two parallel lines and the AC component the common slope of the lines. The second configuration in Table 2 is such that the estimated difference in levels and the estimated common

slope are both of practical significance. Here again, the C1-C2 contrast would be described at the simple simple main effect level with no simplification.

Insert Table 2 about here

In the third configuration of Table 2, however, the BC component is estimated to be zero for all practical purposes as reflected by the nearly coincident lines and the C1-C2 contrast can be described at the simple main effect level as shown in Table 2 by averaging over levels of B. The cell equations in Table 1 can be used to obtain $\mu_{i.1} - \mu_{i.2} = [(\beta_3 - \beta_4) + (\beta_5 - \beta_6)/2] + [(\beta_8 - \beta_9) + (\beta_{10} - \beta_{11})/2]A$; this single line and the associated confidence band represents a more parsimonious description than the original two lines for the two levels of B. The fourth configuration of Table 2 exhibits a difference in levels but an estimated zero common slope (i.e., the AC component = 0 and the BC component $\neq$ 0). The contrast description is again at the simple main effect level where A in each of the two simple simple main effect equations for B1 and B2 is set equal to the aptitude grand mean, $\bar{A}$. Thus, the original two lines have been reduced to two points and their associated intervals. Finally, the maximum simplification of description is possible in the last configuration of Table 2 where the estimated lines are coincident with zero common slope. The contrast is described at the main effect level using the equation above with $A = \bar{A}$; that is, the original two lines have simplified to a single point and interval. This same approach

would be used to attempt to simplify the description of other contrasts of interest.

One possible variation on this approach would add a second condition for simplification. If an estimated interaction contrast were approximately zero and if the associated confidence interval were narrow enough to allow confidence that the interaction is zero for all practical purposes in the population, the appropriate simplification would be made. The second condition is very stringent since narrow interval widths for interactions will only be achieved with relatively large studies, especially if the class is the appropriate unit of analysis. Certainly, a researcher would have more confidence that any simplifications reflect the true relationship when they are made on the basis of both of the conditions.

BC Interaction Format. The "ATI format" used above for description clarifies the nature of the AC and AB interactions, but the BC interaction does not emerge clearly. The interaction between the two nominal variables can be described more clearly with a format analogous to that used with three-way ANOVA. Figure 6 illustrates this format with two patterns, each pattern containing the μ's for the six treatment cells at a specific aptitude. Each configuration can be viewed as the result of a vertical "cut" through the pattern of Figure 1. The variation of the BC interaction pattern across different aptitude values reflects the three-way ABC interaction. Confidence intervals on pairwise and interaction contrasts within Figure 5 would be used as before to identify

reliable features of the patterns. Any simplifications based on the analysis associated with Table 2 would also be reflected in Figure 6.

Insert Figure 6 about here

Finally, a researcher may also be interested in the interaction of aptitude and treatment, ignoring the factorial structure of the two treatment factors. That is, it may be important to determine how the relative superiority of each of the six treatments varies with aptitude. This question is answered with point and interval estimates based on the rankings shown in Figure 1. For example, treatment 12 is estimated to be superior to all of the others up to an aptitude of five, while treatment 21 is superior for higher aptitudes. Thus, the $S_{21} - S_{12}$ and $\mu_{121} - \mu_{112}$ contrasts would be among those of interest.

Discussion

A feature of the ATI analysis illustrated above is the use throughout of a model including all possible interaction terms, even when the final ATI description can be simplified. When the number of independent variables is relatively small, such a fully saturated model is manageable and can be used to ensure the best linear fit of the data in each of the cells formed by the nominal factors. Note that this approach may result in a final difference in the level of complexity of the description and that for the model. This difference is analogous to that in nonorthogonal factorial ANOVA (e.g., Blair and Higgins, 1978) where, in the absence of significant interactions, the full interactive model is still used to test and describe the main effects. Such a difference is

not troublesome as long as the model is viewed simply as a tool to arrive at the primary result, the ATI description. If, however, the model itself is to be presented as a final result to help describe ATI relationships, the researcher would want to consider the elimination of model terms based on judgments such as those described in the previous section. Some care would be required, however, in such a simplification of the model. For example, two possible three-way interaction contrasts of interest in the 2x3 ATI design are equal to β_{10} and β_{11}, the two ABC interaction coefficients. Each of these coefficients in a study may be judged to be small enough separately to be of little practical significance. If the coefficients are of opposite sign, however, it is possible that a third three-way interaction contrast which is equal to $\beta_{10} - \beta_{11}$ may be of practical importance, indicating the X_{10} and X_{11} terms should still be retained.

When a researcher wishes to consider simultaneously a larger number of independent variables than illustrated with the current example, an approach based on specification of a fully saturated model would quickly become overwhelming. An obvious modification would be to start with a model containing, say, only the two-way interactions, judge the resulting fit of the model based on residual plots and, if necessary, add curvilinear or higher order interaction terms to adequately fit the data in all cells resulting from the nominal independent variables. A final check on the model derived from such a process might involve the comparison of the resulting estimated regression lines or planes with those from a model with an additional level of interaction terms. Any important

difference in the comparison would suggest the need for further elaboration of the model.

A more fundamental modification, reflecting current practice, would consist of determining a final model for description containing only terms which are statistically significant. The extent of hypothesis testing in such an approach may vary widely, ranging from a single global ATI test to very extensive testing of hypotheses corresponding to simple interactions and simple simple main effects. All hypotheses of potential interest can be constructed from contrasts like those considered in this paper. Most hypotheses could be tested using a partial F statistic based on multiple regression results with forced order of entry (e.g., Kleinbaum and Kupper, 1978, pp. 141-143), and all other hypotheses could be tested using the general linear hypothesis (e.g., Graybill, 1976, pp. 183-192). Interval estimation and multiple comparison testing could then be used for follow-up description for those hypotheses which are rejected.

A model development based on hypothesis testing would be expected to yield different conclusions from the approach stressing description illustrated in this paper. In studies of moderate size, the hypothesis testing approach may yield oversimplified models and descriptions due to the lack of power in detecting interactions of practical significance. On the other hand, the descriptive approach would tend to produce complex models with some terms that may be due only to sampling error. The latter approach would appear to be the more fruitful for the study of ATI's when the researcher accepts the importance of quasi-replications of studies. Cook and Campbell (1979) discuss the role of multiple studies

of a relationship in which treatments, settings, and populations are varied somewhat to determine the limits of generalization for the validity of a relationship. Such a series of studies would allow the researcher to identify those features of a detailed ATI description which are consistently found across modest variations of study conditions. On the other hand, a series of oversimplified descriptions based on models using hypothesis testing would appear to hold less potential for ultimate confidence in an ATI relationship. Only when studies are large enough to provide adequate power for detection of interactions would the two approaches yield the same conclusions. Large studies become less feasible, of course, when resources are needed for replications of studies.

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Table 1

Cell Regression Equations

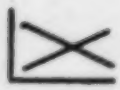
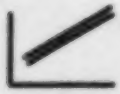
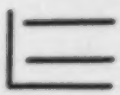
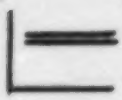
BC cell[a]	Intercept coefficient	Aptitude slope coefficient (S_{jk})
11	$\beta_0 + \beta_2 + \beta_3 + \beta_5$	$\beta_1 + \beta_7 + \beta_8 + \beta_{10}$
12	$\beta_0 + \beta_2 + \beta_4 + \beta_6$	$\beta_1 + \beta_7 + \beta_9 + \beta_{11}$
13	$\beta_0 + \beta_2$	$\beta_1 + \beta_7$
21	$\beta_0 + \beta_3$	$\beta_1 + \beta_8$
22	$\beta_0 + \beta_4$	$\beta_1 + \beta_9$
23	β_0	β_1

[a]First number in the cell designation indicates level of factor B and the second number the level of factor C.

Table 2

Selection of Level of Contrast Description

Based on Interaction Patterns

Pattern configuration[a]	BC interaction component[b]	AC interaction component	C1-C2 contrast description[c]
	NA[d]	NA	$\mu_{ij1} - \mu_{ij2}$
	$\neq 0$	$\neq 0$	$\mu_{ij1} - \mu_{ij2}$
	$= 0$	$\neq 0$	$\mu_{i.1} - \mu_{i.2}$
	$\neq 0$	$= 0$	$\mu_{.j1} - \mu_{.j2}$
	$= 0$	$= 0$	$\mu_{..1} \quad \mu_{..2}$

[a]The vertical axis for each figure is $\gamma = \mu_{ij1} - \mu_{ij2}$, the horizontal axis is aptitude, and the two lines represent the contrast for $j = 1$ and 2.

[b]The BC and AC interaction components are $(\mu_{.11} - \mu_{.12}) - (\mu_{.21} - \mu_{.22})$ and $S(\mu_{i.1} - \mu_{i.2})$ respectively.

[c]The subscripts indicate whether description at the simple simple main effect, simple main effect, or the main effect level is appropriate for the C1-C2 contrast.

[d]Consideration of the global interactions BC and AC is not appropriate when the three-way interaction component of $S(\mu_{111} - \mu_{112}) - S(\mu_{121} - \mu_{122})$ is nonzero.

Figure Captions

Figure 1. Regression equations for BC cells.

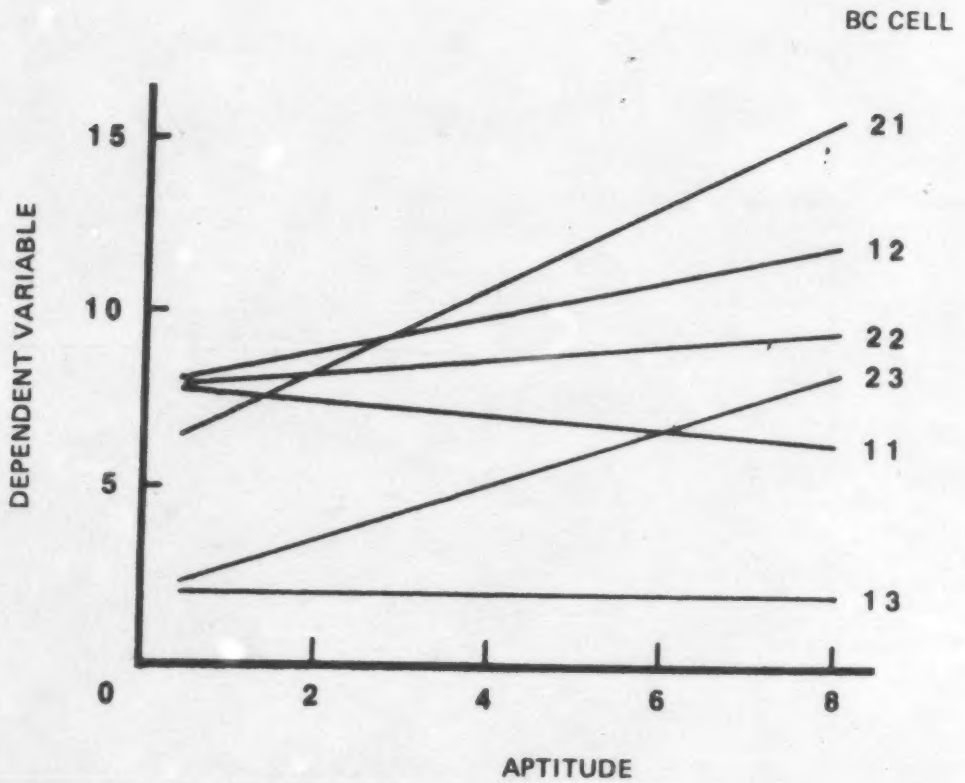
Figure 2. Cell equations for the effect of C at B2.

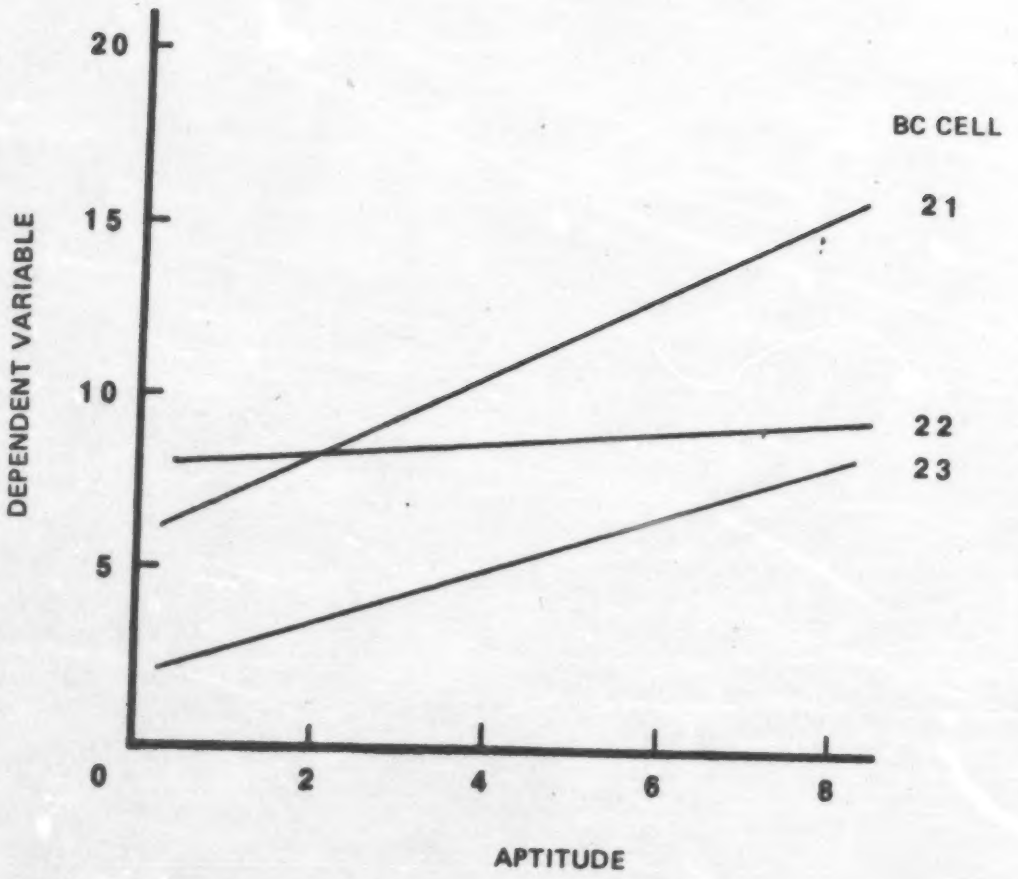
Figure 3. 90% confidence bands on the $\mu_{121} - \mu_{122}$ contrast.

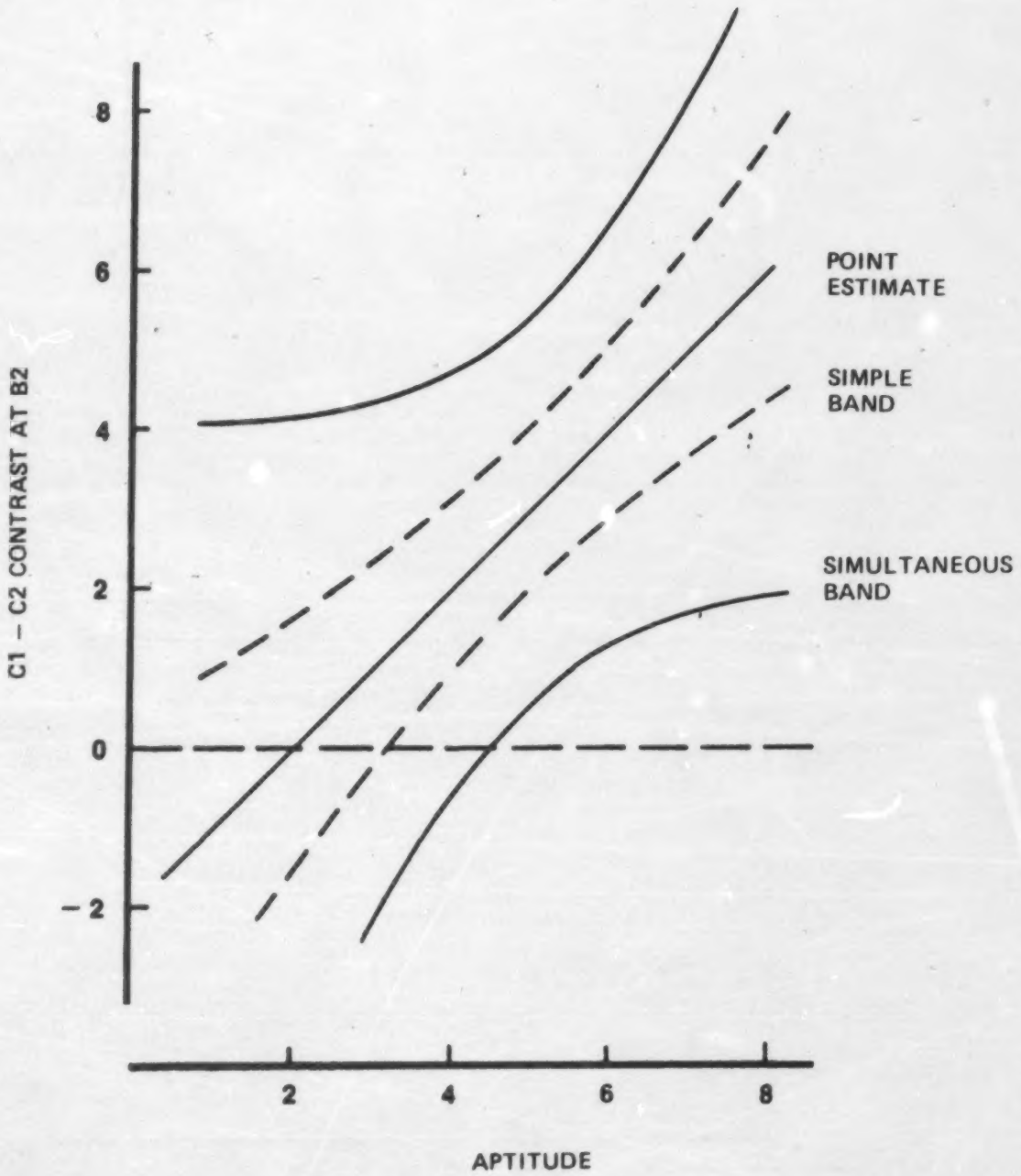
Figure 4. Estimated $\mu_{1j1} - \mu_{1j2}$ contrast for j = 1 and 2.

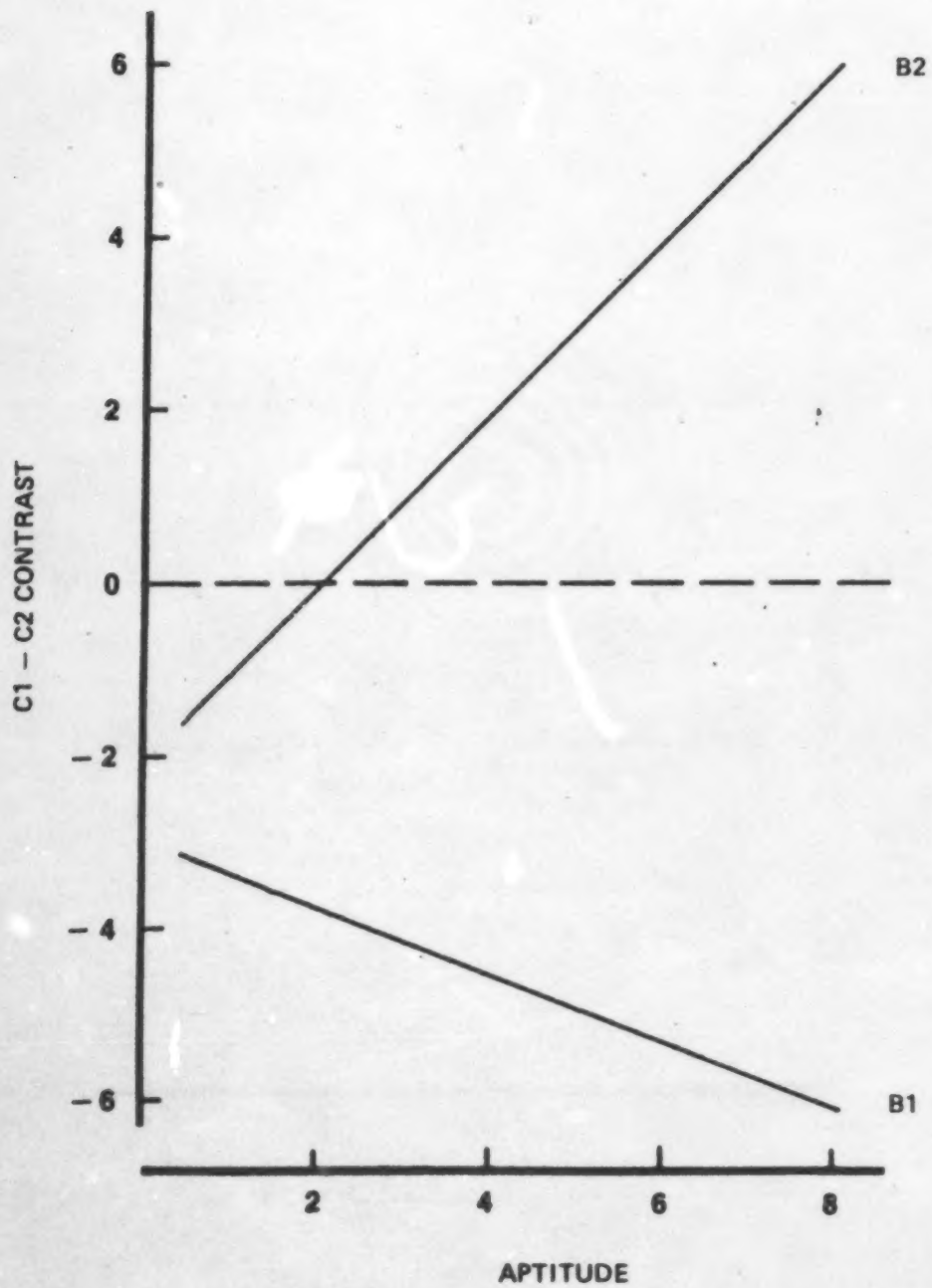
Figure 5. 90% confidence bands on the $(\mu_{111} - \mu_{112}) - (\mu_{121} - \mu_{122})$ interaction contrast.

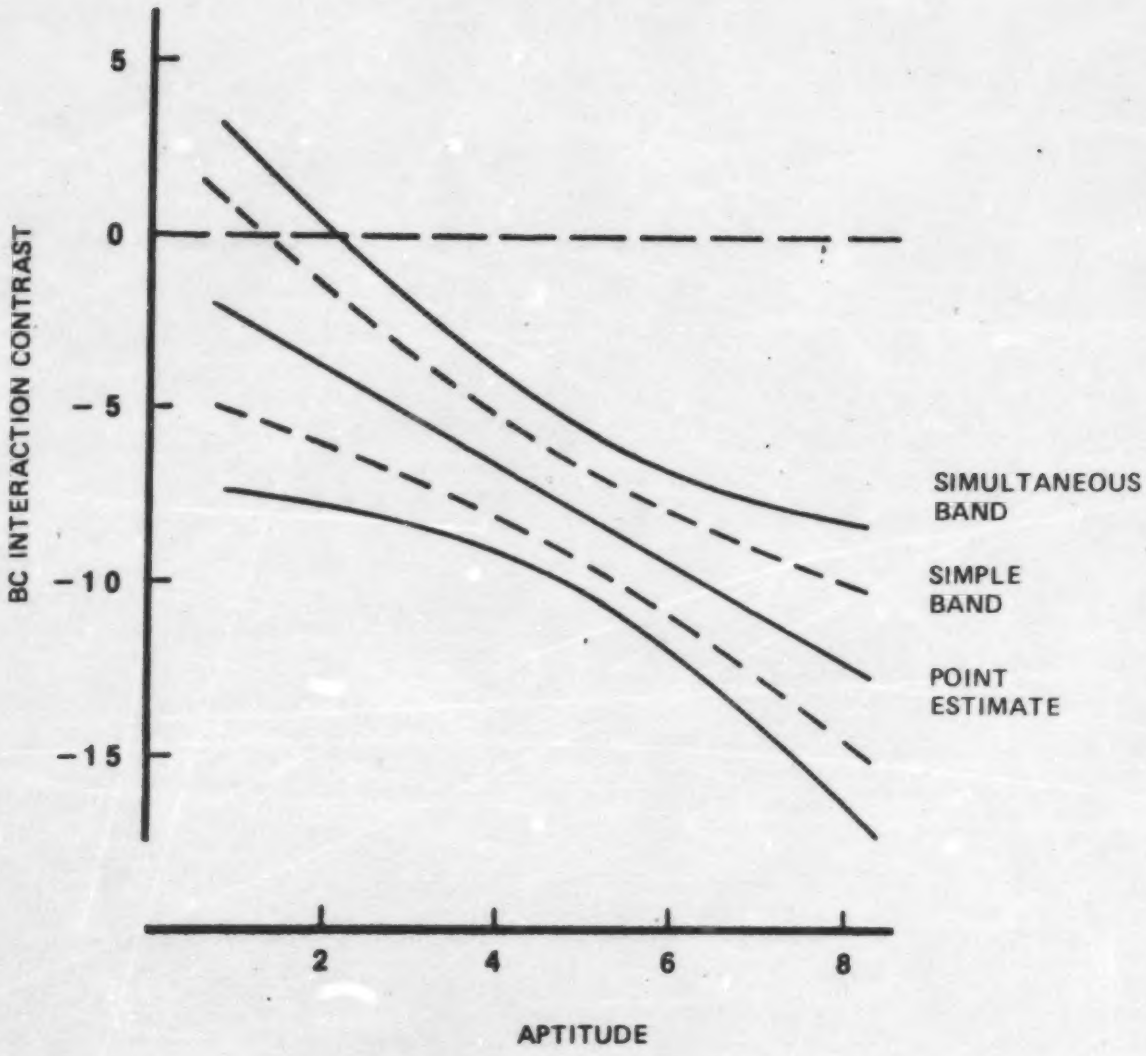
Figure 6. BC interaction pattern for two values of aptitude.

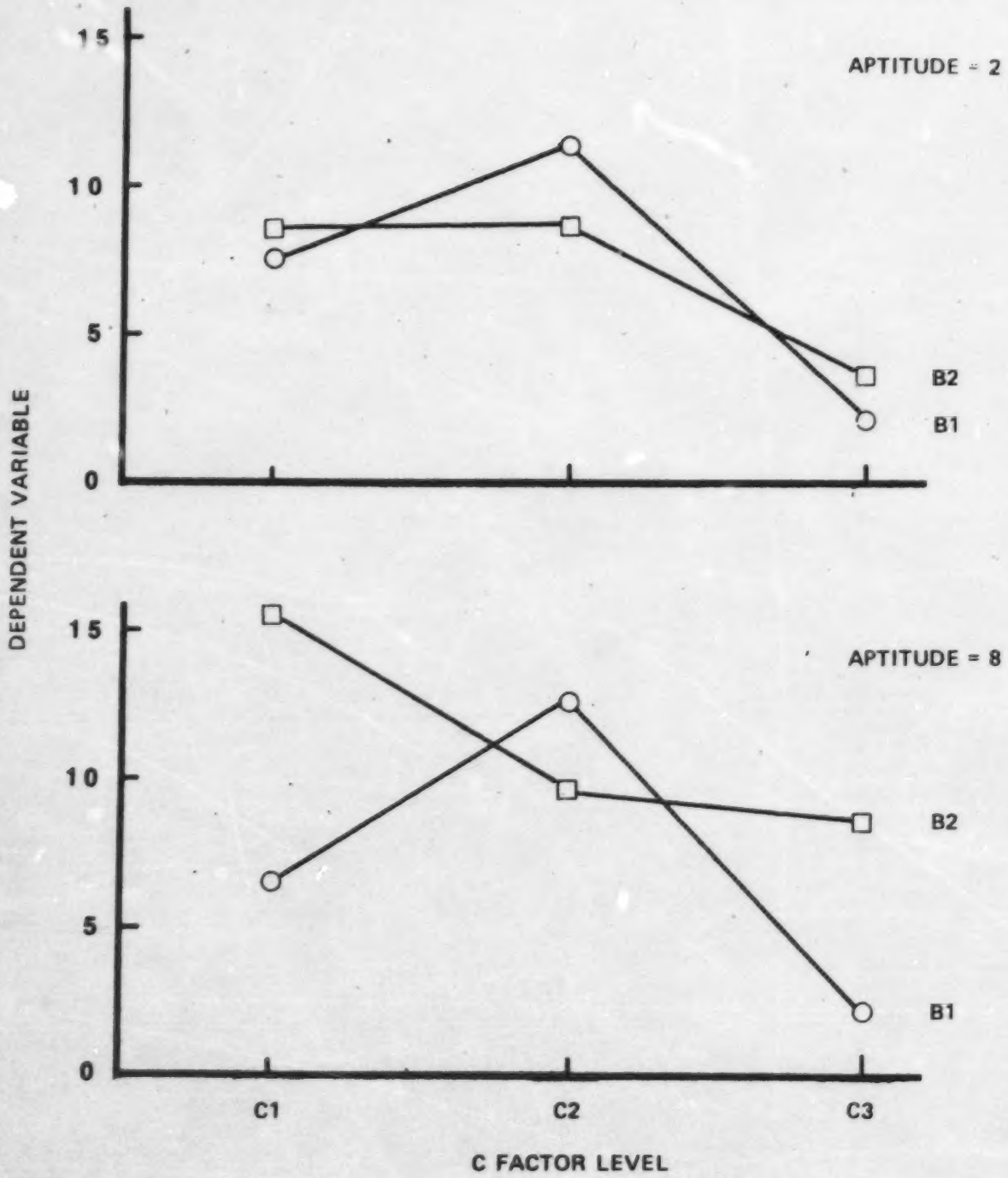

BC CELL
DEPENDENT VARIABLE
15
10
5
0
2
4
6
8
APTITUDE
21
12
22
23
11
13


20
15
10
5
0
2
4
6
8
DEPENDENT VARIABLE
APTITUDE
BC CELL
21
22
23


8
6
4
2
0
−2
C1 − C2 CONTRAST AT B2
2
4
6
8
APTITUDE
POINT ESTIMATE
SIMPLE BAND
SIMULTANEOUS BAND


6
4
2
0
−2
−4
−6
C1 − C2 CONTRAST
B2
B1
2
4
6
8
APTITUDE


5
0
-5
-10
-15
BC INTERACTION CONTRAST
2
4
6
8
APTITUDE
SIMULTANEOUS BAND
SIMPLE BAND
POINT ESTIMATE


APTITUDE = 2
15
10
5
0
B2
B1
APTITUDE = 8
15
10
5
0
B2
B1
DEPENDENT VARIABLE
C1
C2
C3
C FACTOR LEVEL